A Simulation Study to Describe the Distribution of Q
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Introduction
The Q statistic represents the exact upper 90% confidence limit of the 75th percentile. Similar to any statistic, Q is dependent on the data from which it is calculated. The statistic Q assumes that the underlying distribution of the data is log-normally distributed, and hence the mean and standard deviation are also assumed to be good estimates of the true distribution of lead levels found leaching in faucets.

In order to have a good estimate of the mean and standard deviation of a population (or product line of faucets, in this case), the sample must be random and representative of the product line. A minimum of three faucets are sampled from the product line. How representative are three faucets of an entire product line? What can we truly conclude based on these faucets? Is what we observed chance or a very good estimate of the truth?

In order to answer the above questions we considered a simulation study. This simulation study will try to explain the distribution of Q, how confident we should be in the results of Q, and if increasing sample size would improve Q.

Simulation Analyses
The Q statistic is sensitive to the mean, standard deviation and therefore also the number of faucets that are sampled. The present issue with the Q statistic is the risk of rejecting a product line (or population of faucets) that should have been accepted. In the simulation study we generated various populations or product lines of faucets with different characteristics in order to investigate the distribution of Q. In a simulation study we can control extraneous factors that normally we have no control or knowledge over. For example, we can generate a population of faucets that have “good” properties, such as low average levels of lead leaching, small standard deviations, and therefore a low 75th percentile. Intuitively this product line should be accepted. From this population we randomly sample 3 mean levels of lead leaching from faucets, calculate the Q statistic, and determine if it passes (Q<11) or fails (Q>11). This is completed 1000 times from the same population, in order to determine the probability of accepting or rejecting the product line. Similarly, we generate a population of faucets with “bad” properties such as high average levels of lead leaching, and thus a higher 75th percentile. Intuitively, this product line or population of faucets should be rejected. Again, we randomly sample 3 mean levels of lead leaching from faucets from this population or product line, calculate the Q statistic, and determine if it passes (Q<11) or fails (Q>11). This is completed 1000 times from the same population, and the probability of accepting or rejecting the product line is calculated.

In the above simulations we have also calculated the probability of accepting or rejecting the product line under the proposed standard of Q<5.
Description of Simulated Populations
Pete Greiner compiled a list of 317 studies (“Q Basis Comparisons - 2-21-08.xls”) where the Q statistic was calculated after 3 days of measurement (days 3, 4, 5), 6 days of measurement (days 3, 4, 5, 10, 11, 12), and the full 9 days of measurement (days 3, 4, 5, 10, 11, 12, 17, 18, 19). Other statistics in the document included the number of faucets used in each respective study, the geometric mean, and standard deviation on the log scale and the exponential of the standard deviation. The mean on the log scale can simply be calculated by taking the log of the geometric mean.

Simulations were considered for data based on results from only the first three days of study, six days of study and the full nine days of study. For each of these data sets the following procedure was carried out in order to determine the cases to be considered in the simulation. These cases also allow for comparisons between results based on 3, 6, and 9 days of measurements.

First all 317 studies were ordered from least to greatest based on the natural log mean of each study. Next the 317 studies were divided into the following four groups, natural log mean less than -2, between -2 and 0, between 0 and 2, and greater than 2. On the original scale of the data these groupings would be less than 0.14, between 0.14 and 1, between 1 and 7.40, and greater than 7.40. Table 1a gives a summary of the proportion of studies that fall into each of these groups based on the number of days used to calculate the Q statistic. It was observed that when more days are used in the study there are a greater number of studies that fall in the groups with smaller means of lead leaching. As well, the proportion of studies with log mean greater than 2 decreases as the number of days used in the study increases. (Note: The more days used to calculate the Q statistic then the lower values from days 17, 18 and 19 will be used to calculate the mean, this in turn will decrease the mean. This implies that the more days used in the study, the higher the proportion of studies will fall in the groups with lower lead leaching values.) Table 1a also reports the proportion of studies in each group that had a Q statistic less than 5, again based on the number of days used to calculate the Q statistic.

For each group a weighted mean (on the natural log scale) is calculated, weighted by the number of faucets used in each study. The geometric mean can then be calculated by simply taking the exponential of this weighted mean. This weighted mean is used as the mean to simulate the population of faucets for each group. Table 1b reports the weighted mean on the natural log scale for each group (along with the geometric mean), based on the number of days used to calculate the Q statistic. As the number of days used to measure lead leaching increases, the mean of lead leaching decreases. Table 1b also reports the range of the standard deviations for each group, based on the number of days used to calculate the Q statistic. The median and the midpoint of this range are used as the standard deviation for the simulated populations. (Note: a pooled standard deviation was not used as this would assume that the standard deviation from each study within a group was statistically similar, which is not the case.)
The distribution of standard deviations for each group was right skewed. The median of the standard deviations in each grouping represents the 50\(^{th}\) percentile of the standard deviations. The median, although representative of the bulk of the data, did not capture scenarios where the standard deviation was larger. It is these cases where the standard deviation is larger that cause greater concern. That is to say, product lines that should be accepted, but because the standard deviation between 3 faucets is large the Q statistic does not meet the standard, and therefore the product line is rejected. These are the cases of interest and need further investigation.

If we know the mean (\(\mu\)) and the standard deviation (\(\sigma\)) of the population we can calculate the 75\(^{th}\) percentile (\(x_{75}\)) of the population as follows

\[
\mu + z\sigma = x_{75},
\]

where \(z\) is the corresponding 75\(^{th}\) percentile of the standard normal distribution. The mean (\(\mu\)) and the standard deviation must be on the natural log scale, and therefore \(x_{75}\) will also be on the natural log scale. In order to back transform the 75\(^{th}\) percentile (\(x_{75}\)) we would exponentiate \(x_{75}\).

Results from Simulation Study
The simulation scenarios considered in Tables 2-4 are dependent on the true mean and standard deviation used to describe the product line or population of faucets. These simulations are presented in order to describe the distribution of the Q statistic under different scenarios. As well as to illustrate how the Q statistic (like any other statistic) is dependent on sample size. Again, the results from the simulation study are very dependent on the choice of mean and standard deviation used to describe the population of faucets.

Table 2 presents simulations based on studies with 9 days of measurement. Table 3 presents the simulations based on studies with 3 days of measurement. Finally, Table 4 presents the simulations based on studies with 6 days of measurement. The four cases considered in each table correspond to the four groups of mean ranges on the log scale as presented in Table 1a and 1b. For each case considered there is a case #a, and #b. Cases #a indicate that the median of the standard deviations was used to simulate a population of faucets. Cases #b indicate that the midpoint of the range of the standard deviations was used to simulate a population of faucets. Cases #b represent scenarios with larger standard deviations, since the midpoint was usually three times larger than the median.

In all three tables (Tables 2-4) the column P(X<11) indicates the proportion of faucets in the product line (or population) that have average levels of lead leaching less than 11. Similarly, the column P(X<5) indicates the proportion of faucets in the product line (or population) that have average levels of lead leaching less than 5. Finally, columns P(Q<11) and P(Q<5) are the proportion of Q statistics that are less than 11 and 5 respectively, if \(n\) faucets are sampled from the product line.
In all three tables (Tables 2-4) cases 1a, 1b represent studies with very low levels of lead leaching (average mean on the natural log scale was less than -2). These studies will generally have very low 75th percentiles, therefore the Q statistic will likely always pass even with only 3 faucets being sampled. Although these studies only represent 12-14% of what was actually seen among the 317 studies. Cases 4a, 4b (in all three tables) represent studies where the faucets should fail due to very high levels of lead leaching and therefore higher 75th percentiles. When only 3 days of measurements are considered these studies represent 10% of the 317 studies. This proportion decreases to 4% if 9 days of measurements are considered in the calculation of the Q statistic. The bulk of the studies are represented by simulation cases 2a/2b and 3a/3b. Case 2a/2b represented 30-35% of the 317 studies, and case 3a/3b represented 46-48% of the studies.

Results are similar for case 2a whether we test for 3, 6 or 9 days. These cases have low average lead leaching (0.46) and small standard deviations (0.23, 0.19, and 0.17, respectively). Even if only 3 faucets are sampled the product line will pass. If the standard deviation is larger, as seen in case 2b for 3, 6, or 9 days it is recommended that a minimum of 5 faucets be used in order to meet the new standard of Q<5.

Let’s consider case 3a/3b, since this is the most indicative of cases observed among the 317 studies and the one to cause the most concern. Studies where two faucets were similar, but one was slightly different fall into this group. These studies would have larger standard deviations and therefore a larger non-passing Q statistic. Case 3a present simulated populations of faucets using the median of the standard deviation for 3, 6 and 9 days of study. If the standard deviation is small (i.e., all faucets sampled are similar to one another), then even when only 3 faucets are sampled on only 3 days is sufficient to give a passing Q but how much information is this really telling us? When looking at very limited data the standard deviation will usually be very small and therefore give a small Q statistic. How informative is this of what might really be there?

Case 3b present simulated populations of faucets using the midpoint of the range of the standard deviation for 3, 6 and 9 days of study. The proportion of faucets with average lead leaching levels less than 11 (P(X<11)) in case 3b increases with the number of days considered in the study. For example, the proportion P(X<11) is 99.5% for only 3 days of study, 99.8% for 6 days of study and 99.99% for the full 9 days of study. Consequently, if only 3 or 6 days will be considered for study, then a minimum of 5 faucets should be sampled in order to meet the current standard of Q<11.

The proportion of faucets with average lead leaching levels less than 5 (P(X<5)) in case 3b increases with the number of days considered in the study. For example, the proportion P(X<5) is 88% for only 3 days of study, 91% for 6 days of study and 97% for the full 9 days of study. If only 3 days will be considered for study, then even when 15 faucets are sampled the probability of accepting the product line is only 59% (Table 3). If 6 days will be used for study then a minimum of 15 faucets should be sampled in order to accept the product line 80% of the time (Table 4) under the proposed standard (Q<5). Finally if the full 9 days are used for study then a minimum of 10 faucets should be sampled in order to meet the proposed standard of Q<5 (Table 2).
Conclusions
The results from the simulation study are highly dependent on the choice of mean and standard deviation used to describe the population of faucets. If these choices are representative of what is seen in actual studies then the results of the simulation study are meaningful. The lesson learned is: when the standard deviation is small (assuming reasonable average levels of lead leaching from faucets) usually the Q statistic will meet the standard. This is indicative of cases where the faucets are behaving similarly, thus giving a small standard deviation. In these cases 3 faucets is sufficient. Conversely, when the standard deviation is larger, the probability of accepting Q is lowered. As well when fewer sample points are taken per faucet tested, a greater numbers of faucets will require testing to demonstrate compliance with the standard.

Table 1a presented the proportion of studies in each group considered that had studies with passing Q’s under the proposed standard (Q<5) for 3, 6 and 9 days of study. Group 3 (with average geometric mean of lead leaching levels between 1 and 7.4µg) after 9 days of study only 71% of these would have passed the new standard. Of these, 74% had more than 3 faucets included in the study. If only 3 days of study would have been considered then only 63% of this groups studies would have passed (37% would have failed). Of the 63% that passed, 76% of them included more than 3 faucets.

The Q statistic is calculating exactly what it was set out to calculate (i.e., the upper 90% confidence limit of the 75th percentile). The Q statistic, like any other statistic, is as good as the data that is used to compute its value. The greater the number of faucets used in the study, the more accurate and representative the estimate of the standard deviation and the mean. The more accurate and representative the mean and standard deviation, then the more confidence we have in the Q statistic.

Based on the summary statistics of the 317 studies in Table 1a it is recommended that a minimum of 6 days should be considered in the study. As for the number of faucets to be used it will depend on the types of assumptions we are willing to make about the population of faucets or a product line. If we want to assume that standard deviations will always be small (similar to case 3a in Tables 2-4) then 3 faucets would be sufficient. But these are not the cases that cause us concern or that have brought about this exercise. The cases with large standard deviations from a product line that should have passed are the ones that cause concern (cases 3b in Tables 2-4). If the objective is to overcome this problem then 15 faucets should be considered with only 6 days of study in order to pass 80% of product lines. Perhaps this scenario is not often seen, in which case it is not cause for concern and this exercise was simply to investigate the properties of Q and determine if it still meets the needs of its users. If that is the case, the answer is Q is doing exactly what it was set out to do.
Table 1a: Summary of 317 studies used for simulation for 3 days, 6 days and 9 days

<table>
<thead>
<tr>
<th>Mean ranges on natural log scale and original scale</th>
<th>Percent of studies represented among the 317 studies</th>
<th>Percent of studies with Q&lt;5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 days</td>
<td>6 days</td>
</tr>
<tr>
<td>1.  &lt; -2 ( &lt;0.14)</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>2.  (-2, 0) (0.14, 1)</td>
<td>30</td>
<td>34</td>
</tr>
<tr>
<td>3.  (0, 2) (1, 7.40)</td>
<td>48</td>
<td>46</td>
</tr>
<tr>
<td>4.  &gt; 2 (&gt; 7.40)</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1b: Summary of 317 studies used for simulation for 3 days, 6 days and 9 days

<table>
<thead>
<tr>
<th>Mean ranges on natural log scale and original scale</th>
<th>Weighted mean on natural log scale (geometric weighted mean)*</th>
<th>Range of standard deviation on natural log scale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 days</td>
<td>6 days</td>
</tr>
<tr>
<td>1.  &lt; -2 ( &lt;0.14)</td>
<td>-3.02</td>
<td>-3.03</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>2.  (-2, 0) (0.14, 1)</td>
<td>-0.77</td>
<td>-0.76</td>
</tr>
<tr>
<td></td>
<td>(0.46)</td>
<td>(0.47)</td>
</tr>
<tr>
<td>3.  (0, 2) (1, 7.40)</td>
<td>0.95</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>(2.59)</td>
<td>(2.50)</td>
</tr>
<tr>
<td>4.  &gt; 2 (&gt; 7.40)</td>
<td>2.49</td>
<td>2.36</td>
</tr>
<tr>
<td></td>
<td>(12.04)</td>
<td>(10.61)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Means were weighted by the number of faucets used in each study
Table 2: Simulations based on typical lead leaching from faucets after all 9 days (days 3, 4, 5, 10, 11, 12, 17, 18 and 19)

<table>
<thead>
<tr>
<th>Cases Considered</th>
<th>n=3</th>
<th>P(X&lt;11)*</th>
<th>P(Q&lt;11)**</th>
<th>P(X&lt;5)*</th>
<th>P(Q&lt;5)**</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a. $x_{75} = 0.056$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\exp(\mu) = 0.05$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\mu = -2.97$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\sigma = 0.12$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1b. $x_{75} = 0.065$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\exp(\mu) = 0.05$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\mu = -2.97$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\sigma = 0.34$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2a. $x_{75} = 0.50$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\exp(\mu) = 0.44$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\mu = -0.82$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\sigma = 0.17$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2b. $x_{75} = 0.75$</td>
<td></td>
<td>99.99%</td>
<td>90.5%</td>
<td>99.9%</td>
<td>72.6%</td>
</tr>
<tr>
<td>$\exp(\mu) = 0.44$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\mu = -0.82$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\sigma = 0.78$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3a. $x_{75} = 2.53$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>99.99%</td>
<td>96.3%</td>
</tr>
<tr>
<td>$\exp(\mu) = 2.28$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\mu = 0.82$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\sigma = 0.16$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3b. $x_{75} = 2.99$</td>
<td></td>
<td>99.99%</td>
<td>87.4%</td>
<td>97%</td>
<td>40.6%</td>
</tr>
<tr>
<td>$\exp(\mu) = 2.28$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\mu = 0.82$</td>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>$\sigma = 0.41$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4a. $x_{75} = 10.49$</td>
<td></td>
<td>85%</td>
<td>18.5%</td>
<td>&lt;0.01%</td>
<td>0%</td>
</tr>
<tr>
<td>$\exp(\mu) = 9.63$</td>
<td></td>
<td>23.3%</td>
<td>34.8%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$\mu = 2.26$</td>
<td></td>
<td>47.3%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$\sigma = 0.13$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4b. $x_{75} = 10.53$</td>
<td></td>
<td>84%</td>
<td>15.7%</td>
<td>&lt;0.01%</td>
<td>0%</td>
</tr>
<tr>
<td>$\exp(\mu) = 9.63$</td>
<td></td>
<td>20.8%</td>
<td>27.7%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$\mu = 2.26$</td>
<td></td>
<td>37.7%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$\sigma = 0.14$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

= $\mu$ and $\sigma$ are computed on the log scale of lead leaching from faucets. Therefore $\exp(\mu)$ is the geometric mean and $x_{75}$ is the 75th percentile on the original scale of the data.

* Proportion of faucets in product line having lead values less than 11 or 5$\mu$g

**Proportion of samples of size n from the product line that would yield a passing Q statistic
Table 3: Simulations based on typical lead leaching from faucets after only 3 days (days 3, 4, and 5)

<table>
<thead>
<tr>
<th>Cases Considered</th>
<th>P(X&lt;11)*</th>
<th>P(Q&lt;11)**</th>
<th>P(X&lt;5)*</th>
<th>P(Q&lt;5)**</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a. $x_{75} = 0.053$</td>
<td>n=3 100%</td>
<td>n=5 100%</td>
<td>n=10 100%</td>
<td>n=15 100%</td>
</tr>
<tr>
<td>exp($\mu$) = 0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = -3.02$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.13$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1b. $x_{75} = 0.061$</td>
<td>n=3 100%</td>
<td>n=5 100%</td>
<td>n=10 100%</td>
<td>n=15 100%</td>
</tr>
<tr>
<td>exp($\mu$) = 0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = -3.02$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.34$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2a. $x_{75} = 0.54$</td>
<td>n=3 100%</td>
<td>n=5 100%</td>
<td>n=10 100%</td>
<td>n=15 100%</td>
</tr>
<tr>
<td>exp($\mu$) = 0.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = -0.77$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.23$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2b. $x_{75} = 0.72$</td>
<td>n=3 99.99%</td>
<td>n=5 99.6%</td>
<td>n=10 99.98%</td>
<td>n=15 83.5%</td>
</tr>
<tr>
<td>exp($\mu$) = 0.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = -0.77$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.66$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3a. $x_{75} = 2.94$</td>
<td>n=3 100%</td>
<td>n=5 100%</td>
<td>n=10 99.97%</td>
<td>n=15 85.1%</td>
</tr>
<tr>
<td>exp($\mu$) = 2.59</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.95$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.18$</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>3b. $x_{75} = 3.77$</td>
<td>n=3 99.5%</td>
<td>n=5 59.6%</td>
<td>n=10 88%</td>
<td>n=15 20.2%</td>
</tr>
<tr>
<td>exp($\mu$) = 2.59</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.95$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.56$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4a. $x_{75} = 13.71$</td>
<td>n=3 31%</td>
<td>n=5 0.3%</td>
<td>n=10 &lt;0.01%</td>
<td>n=15 0%</td>
</tr>
<tr>
<td>exp($\mu$) = 12.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 2.49$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.19$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4b. $x_{75} = 14.28$</td>
<td>n=3 36%</td>
<td>n=5 0.9%</td>
<td>n=10 &lt;0.02%</td>
<td>n=15 0%</td>
</tr>
<tr>
<td>exp($\mu$) = 12.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 2.49$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.25$</td>
<td></td>
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</tr>
</tbody>
</table>

= $\mu$ and $\sigma$ are computed on the log scale of lead leaching from faucets. Therefore exp($\mu$) is the geometric mean and $x_{75}$ is the 75th percentile on the original scale of the data.

* Proportion of faucets in product line having lead values less than 11 or 5\(\mu\)g

**Proportion of samples of size n from the product line that would yield a passing Q statistic
Table 4: Simulations based on typical lead leaching from faucets after only 6 days (days 3, 4, 5, 10, 11, and 12)

<table>
<thead>
<tr>
<th>Cases Considered</th>
<th>P(X&lt;11)*</th>
<th>P(Q&lt;11)**</th>
<th>P(X&lt;5)*</th>
<th>P(Q&lt;5)**</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1a.</strong> $x_{75} = 0.052$</td>
<td>n=3 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>$\exp(\mu) = 0.05$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mu = -3.03$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.11$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>n=5 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>n=10 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>n=15 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>1b.</strong> $x_{75} = 0.060$</td>
<td>n=3 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>$\exp(\mu) = 0.05$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mu = -3.03$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.33$</td>
<td></td>
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<tr>
<td></td>
<td>n=5 100%</td>
<td>100%</td>
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<td></td>
<td>n=10 100%</td>
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<tr>
<td></td>
<td>n=15 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>2a.</strong> $x_{75} = 0.53$</td>
<td>n=3 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>$\exp(\mu) = 0.47$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mu = -0.76$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.19$</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>n=5 100%</td>
<td>100%</td>
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<td>n=10 100%</td>
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<tr>
<td></td>
<td>n=15 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>2b.</strong> $x_{75} = 0.77$</td>
<td>n=3 99.99%</td>
<td>92.2%</td>
<td>99.9%</td>
<td>77.5%</td>
</tr>
<tr>
<td></td>
<td>$\exp(\mu) = 0.47$</td>
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<tr>
<td></td>
<td>$\mu = -0.76$</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$\sigma = 0.73$</td>
<td></td>
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<tr>
<td></td>
<td>n=5 100%</td>
<td>100%</td>
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<td>n=10 100%</td>
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<td></td>
<td>n=15 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>3a.</strong> $x_{75} = 2.79$</td>
<td>n=3 100%</td>
<td>100%</td>
<td>99.98%</td>
<td>90.1%</td>
</tr>
<tr>
<td></td>
<td>$\exp(\mu) = 2.50$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>$\mu = 0.91$</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$\sigma = 0.17$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>n=5 100%</td>
<td>100%</td>
<td>99.9%</td>
<td>99.9%</td>
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<td></td>
<td>n=10 100%</td>
<td>100%</td>
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<td></td>
<td>n=15 100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>3b.</strong> $x_{75} = 3.50$</td>
<td>n=3 99.8%</td>
<td>68.7%</td>
<td>91%</td>
<td>25.6%</td>
</tr>
<tr>
<td></td>
<td>$\exp(\mu) = 2.50$</td>
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</tr>
<tr>
<td></td>
<td>$\mu = 0.91$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.51$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>n=5 96.4%</td>
<td>100%</td>
<td>39.6%</td>
<td>62.8%</td>
</tr>
<tr>
<td></td>
<td>n=10 100%</td>
<td>100%</td>
<td>62.8%</td>
<td>80.6%</td>
</tr>
<tr>
<td></td>
<td>n=15 100%</td>
<td>100%</td>
<td>62.8%</td>
<td>80.6%</td>
</tr>
<tr>
<td><strong>4a.</strong> $x_{75} = 11.80$</td>
<td>n=3 59%</td>
<td>2.9%</td>
<td>&lt;0.01%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>$\exp(\mu) = 10.61$</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$\mu = 2.36$</td>
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<tr>
<td></td>
<td>$\sigma = 0.16$</td>
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<tr>
<td></td>
<td>n=5 2.2%</td>
<td>2.2%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>n=10 1.2%</td>
<td>1.2%</td>
<td>0%</td>
<td>0%</td>
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<tr>
<td></td>
<td>n=15 0.2%</td>
<td>0.2%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td><strong>4b.</strong> $x_{75} = 12.62$</td>
<td>n=3 56%</td>
<td>3.2%</td>
<td>0.2%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>$\exp(\mu) = 10.61$</td>
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<td>$\mu = 2.36$</td>
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<tr>
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<td>$\sigma = 0.26$</td>
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</tr>
<tr>
<td></td>
<td>n=5 1.6%</td>
<td>1.6%</td>
<td>0%</td>
<td>0%</td>
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<tr>
<td></td>
<td>n=10 0.7%</td>
<td>0.7%</td>
<td>0%</td>
<td>0%</td>
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<tr>
<td></td>
<td>n=15 0.2%</td>
<td>0.2%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

= \mu and \sigma are computed on the log scale of lead leaching from faucets. Therefore \exp(\mu) is the geometric mean and $x_{75}$ is the 75th percentile on the original scale of the data.

* Proportion of faucets in product line having lead values less than 11 or 5 \mu g

**Proportion of samples of size n from the product line that would yield a passing Q statistic